No. XLIX.

Description and use of a new and simple Nautical Chart, for working the different problems in Navigation; with examples of its application according to Mercator's Sailing, and sailing by the Arc of a Great Circle; with a demonstration of its principles. By John Garnett, of New Brunswick, New Jersey,

"*Segnius irritant animos demissa per aurem,*
"*Quàm quæ sunt oculis subjecta fidelibus, et quæ*
"*Ipse sibi tradit spectator.*"

☞ To the Author of this Communication an **Extra-Magellanic Premium** of a Gold medal was awarded by the Society.

Read August 25th, 1807.

THIS Chart is a partial projection of a portion of a spherical or spheroidal surface, containing in length the different degrees of latitude, and in breadth as many of longitude as are found necessary. The parallels of latitude are projected into right lines, parallel and equidistant to each other, (the reason of which will plainly appear in the demonstration of the principle at the end;) and are divided into degrees of longitude by a scale of equal parts, according to the length of each degree at different latitudes, either in the sphere or spheroid, or by Table *T* of the Requisite Tables.*—Through these divisions the different meridians are drawn or engraved, on both sides the *Central Meridian*, which is always a right line.—*Vide chart.*

The Index (which is separate) contains all the courses in the quadrant of a circle, both in degrees, and quarter points, and also the distance sailed as far as necessary; in using it, the chief attention requisite is *to place the center* C *of the quadrant on that point in any given parallel of latitude, from which* the distance sailed shall subtend *nearly*† *an equal difference of longitude on each side the Central Meridian* or middle line of

* Garnett's Requisite Tables.

† It is not necessary that it should be exactly so, as the beginning of the distance, or Center *C*, had better be placed on an engraved meridian, so that the difference of longitude may be seen at one view at the other extremity of the distance.

the Chart, and that the given parallel of latitude shall cut the course on the Index, either in degrees or points of the compass; extending naturally towards the North or South as the course directs, but indifferently whether to the right or left, as either may occasionally represent the East or West (the difference of longitude being equal on either side,) so that the centre *C* of the Index must be placed on the *right hand* of the centre of the Chart, when the course is *towards* the equator, and on the *left hand* when *from* it, as will readily appear in the practice.

The solutions of all the cases being on a more correct principle than the common method by middle latitude, will be found more accurate—particularly where the difference of latitude is great.

Should the distance exceed the extent of the Index, or the whole difference of longitude on the Chart, the work must be repeated from the last found latitude, until the difference of longitude and latitude corresponding to the whole distance is obtained.

The following examples of the different Cases of Sailing by this Chart, will make its use sufficiently easy.

CASE I.

Given the Latitudes *and* Longitudes *of two places; to find the Course and Distance between them.*

EXAMPLE.

Suppose the Latitudes of the two places to be 49° 10′ N. and 53° 20′ N. respectively, and their difference of Longitude 6° 10′; required the *course* and *distance* between them.

1st. Lay the centre *C* of the Index on the meridian which is about half the given difference of longitude, or 3 degrees on the *left side* of the Central Meridian, and in the parallel of latitude 49° 10′.

2nd. Extend the distance line of the Index to the parallel of 53° 20′ of latitude, on the meridian of 3° 10′ diff. of long. at the *right side* of the Central Meridian,—and the distance

found on the Index will be 340 miles;—and the course crossed by the parallel of latitude, will be N. 42 3-4 degrees, either East or West; according as the latter place is Eastward or Westward of the former. Q. E. I.

N. B. When the differences of latitude and longitude are great, as it often happens in this Case, the course and distance may be found sufficiently accurate for practice on the *general* Chart of this projection. But in *Great Circle Sailing* the angles of position should be found by Spherical Trigonometry.

CASE II.

Given one Latitude, Course *and* Distance, *to find the other Latitude, and difference of Longitude.*

EXAMPLE.

A ship from latitude 52° 10′ N. and longitude 35° 6′ West, sails N. W. b. W. 229 miles; required the *latitude* and *longitude* arrived at.

1st. Lay the centre *C* of the Index (to the left side) on the parallel of latitude 50° 10′ and turn it about until the parallel passes through the 5 point course; then slide it on the parallel until the distance 229 miles subtends nearly an equal difference of longitude on each side the Central Meridian; which will be found 2° 40 on each side, or 5° 20′ diff. of longitude, when the distance will also reach the parallel of latitude 54° 17′; for the latitude arrived at. Q. E. I.

CASE III.

Given both Latitudes *and the* Course; *to find the Distance, and Difference of Longitude.*

EXAMPLE.

A ship sails N. E. b. E. from latitude 42° 25′ N. and longitude 15° 6′ W. and then finds by observation she is in latitude 46° 20′ N.; required the *distance,* and present *longitude.*

N. B. If the distance extend beyond the Chart, which can seldom happen in practice, it will require two operations as in this instance. (Or it may be performed at once on the general Chart.)

1st. Set the center *C* of the Index on the *left side* of the Central Meridian (because sailing *from* the Equator) to 2° 30′ diff. of longitude, and on the given parallel of latitude 42° 25′.

2nd, Extend the distance line on a 5 point course, to 2° 30′ diff. of longitude on the *right hand* of the Central Meridian, and the first latitude will be found 44° 48′ N. diff. of longitude 5° East; and distance 258 miles; which write down as under, for the first operation. Then set the Index *C* to the last found latitude, and the distance line on a 5 point course will extend from 1° 40′ diff. longitude on each side, to the given latitude of 46° 20′, and measure 167 miles; which added as under to the first found distance and difference of longitude, gives the whole distance and difference of longitude.

Lat. sailed from 42° 25′ longit. 15° 6′ W. course N. 5 pts. E.

To latit. 44 48 diff. long. 5 0 E. distance 258 miles,
To latit. 46 20 diff. long. 3 20 E. distance 167 miles.

Gives the required diff. longitude 8° 20′ E and dist. 425 miles.

CASE IV.

Given the Latitudes *of two Places, and the* Distance *between them to find the Course and Difference of Longitude.*

EXAMPLE.

A ship from St. Alban's Head, in latitude 50° 35′ N. and longitude 2° 5′ W. sailed 171 miles upon a direct course between the S. and W. and by observation is found to be in latitude 48° 26′ N. required the *course* steered, and *longitude* come to?

1st. Set the center *C* of the Index on the given parallel of latitude 50° 35′ (on the *right hand*, because sailing towards the Equator) and turn the distance line until the given distance 171

miles falls on the parallel of 48° 26′, *extended equally on each side the central line;* when the course will be found S. 41° W. and the difference of longitude 2° 50′. Q. E. I.

CASE V.

Given one Latitude, Course, *and* Difference of Longitude, *to find the other Longitude, and distance.*

EXAMPLE.

A ship from latitude 47° 30′ N. sails S. 51° W. and then finds her difference of longitude by observation to be 9° 40′ W. required the *distance* run, and the *latitude* come to?

N. B. As this difference of longitude exceeds the extent of the chart; it requires, (unless performed by the general Chart,) a double operation.

1st. Set the index *C* on the given parallel of 47° 30′ at the *right hand* of the Central Meridian, (sailing towards the Equator) and the given course 51° will give the distance 264 miles, and latitude 44° 44′ at 5° difference of longitude; (2° 30′ on each side of the central meridian.)

2nd. Set the Index on the last found parallel of latitude 44° 44′, and to the same course; when the distance corresponding to 4° 40′ (the remainder of the difference of longitude) setting 2° 20′ on each side the central meridian, will give a farther distance of 265 miles, making the whole distance 529 miles, and the latitude come to 41° 57′. Q. E. I.

* CASE VI.

Given one Latitude, Distance *and* Difference of Longitude, *to find the other Latitude and Course.*

EXAMPLE.

A ship sails from the latitude of 50° 20′ N. between the North and East, 300 miles, and finds by a chronometer her

* This is Dr. Halley's celebrated problem. See Baron Masseres' "Scriptores Logarithmici." vol 4th. *It may be seen on the general chart, that this problem will sometimes admit of two answers.*

difference of longitude to be 6° 0′ required the *latitude* arrived at, and the *course* the vessel has steered?

1st, Place the Index to the given latitude, and on the *left hand* on 3 degrees, (the half of the given difference of longitude) and extend the given distance 300 miles, to 3° difference of longitude on the *right hand;* then the latitude arrived at will be found 53° 45′ N. and the course N. 47° E. Q. E. I.

CASE VII.

Given the Course, Distance, *and* Difference of Longitude, *to find both Latitudes.*

RULE.

Place the Index on any parallel of latitude to the required course, and if the given distance subtend a less difference of longitude that the given, (always making an equal difference of longitude on both sides the central meridian) move it upwards to a higher latitude, or if it subtend a greater difference of longitude, move it downwards to a lower latitude until the distance subtend the given difference of longitude, and the required latitudes will be found.

N. B. When the course is on the meridian, this case is indeterminate, and the nearer to the meridian, the less accurate will be the solution.

CASE VIII.

Given the Course, Difference of Latitude, *and* Difference of Longitude, *to find both Latitudes.*

This case is similar to case 7th. using the difference of latitude instead of the distance, which is always known when the course and difference of latitude are given.

N. B. This CASE like the LAST is also indeterminate when the course is on the meridian, and less accurate when near it.

CASE IX.

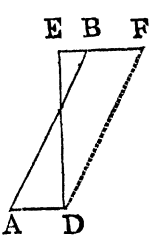
Given the Distance, Difference of Latitude, *and* Difference of Longitude, *to find the Course and both Latitudes*

RULE.

Make the distance on the Index subtend the difference of latitude N. or S. between *any* parallels of latitude, and the Course will be found; and if the difference of longitude on the Chart exceed that given, move the Index to a less latitude: or if it be too little, to a greater latitude, until the given differences of longitudes and latitudes are subtended by the given distance; when both latitudes will be known. Q. E. I.

N. B. This case is indeterminate in the same circumstance, as are the cases 7 and 8, which are only given, to shew that *every* case can be readily solved by this Chart; but the three last cases can seldom be of use in practice.

To demonstrate the principles on which this Chart is constructed, and to shew its application to *Sailing by the Arc of a Great Circle.*



Let A, B, be two places on the Chart, whose difference of longitude is equally divided by the central meridian E D which is the part of it between the two latitudes; draw the line A B; and D F parallel to it; also the parallels of latitude E F, A D; then E F = E B + A D; but E B and A D by the construction of the Chart are = the cosines of their respective latitudes × by half the difference of longitude between A and B; and therefore E F = *half the sum of the cosines* × *by the difference of longitude.*

In the right angled triangle E D F, E F: tangent E D F :: E D: radius. Or *half the sum of the cosines of the latitudes: tangent of the course* :: *difference of latitude: the difference of longitude,* which is a well known theorem in navigation; *half the sum of the cosines of the two latitudes* being generally* more accurate than the *cosine of the middle latitude* commonly used. Q. E. D.

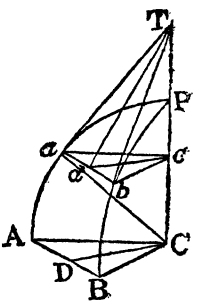
* The exceptions are of no consequence in practice. See Emerson's Navigation, Page 71.

The principles of the Chart, together with the application to *sailing by the Arc of a Great Circle*, can also be deduced from the following propositions.

PROPOSITION I.

The angle of convergence, or inclination of two meridians to each other, at any given latitude, is = *the difference of longitude* × *by the sine of the latitude.*

Let P A, P B, be two meridians, on which let *a*, *b*, be two places in the same latitude; draw the two tangents *a* T, *b* T, meeting the axis C P T in T, and *a c*, *b c*, perpendiculars to it; also *c d*, T *d* perpendiculars to *a b*, and draw *a* C to the center C; then will the angle *a* T *b* represent the inclination of the meridians P *a*, P *b*, to each other at the points *a* and *b*.



From the right angled triangles *a d* T, *a d c* and the similar triangles *a c* T, *a c* C.

a T : rad.:: *a d* : sine $\frac{1}{2}$ inclination of merid. and rad.: *a c*:: sine (*d c a* =) $\frac{1}{2}$ diff. of lon. : *a d*. By composition. (*a* T : *a c* =) *a* C : C *c* :: sine $\frac{1}{2}$ difference of longitude : sine $\frac{1}{2}$ the inclination of meridians; that is, *radius* : *sine of the latitude* :: *sine* $\frac{1}{2}$ *difference of longitude* : *sine* $\frac{1}{2}$ *the inclination of meridians.* Or taking the arcs themselves for their sines, which is sufficiently accurate in small arcs, and agrees with the construction of the Chart; *the angle of convergence of any two meridians at a given latitude, is = to the sine of the latitude* × *by the difference of longitude.* Q. E. D.

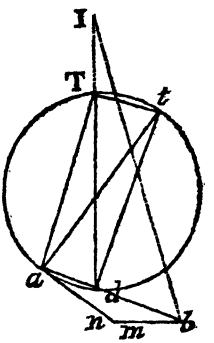
REMARK.

If the meridians be considered as great circles of the sphere, and their inclination to the *central meridian* (or that which bisects the angle at the Pole) as the complement of the angle made by a great circle passing through them at any given latitude, then in the spher. triangle P*ad* we have *rad.*: *cosine* P*a*:: *tang. a* P *d*: *cotan.* P *a d*; *that is, rad.*: *sine of the latitude*:: *tangent of half the difference of longitude* : *tangent of the inclination to the central meridian*; which seems more correct.

See the Table of the Inclination of Meridians, deduced from this Proposition.

PROPOSITION II.

The ANGLE of POSITION at the middle longitude between two places is very nearly equal to the loxodromic angle or course between them; and differs at each place from the loxodromic course, by the inclination of the meridian at each place to the *central meridian.*



Let *a*, *b*, be two places on the arc of a great circle, extended into a right line which is crossed by the central meridian T *d*, and the meridians T *a*, I *b*, at their respective angles of position, then the angles *a* T *d*, *d* I *b* will be the inclination of the meridians at *a* and *b*, to the central meridian, respectively. Draw T *t*, *d t* perpendiculars to *a* T, *a b*, and draw *a t*; then because the loxodromic course, cuts the meridians at equal angles, the course near *a*, may be considered as a portion of the logarithmic spiral; by the known property of which, *t a*, will be the radius of curvature at the point *a*, and *n a* perpendicular to *t a*, its tangent, making the angle *d a n*=*d t a*=*d* T *a*, (being on the same segment *a d* of the circle passing through *a d t* T;) and therefore, the exterior angle T *d b* is=T *a d*+(*d* T *a*=) *d a n*=T *a n*, the course; and the *course must be parallel to a b wherever it crosses the central meridian* T *d*, *otherwise it cannot make an angle with it*=T *a n*.

In the same manner I *b m* may be proved =*d b* I+*d* I *b* =I *d a*. Q. E. D.

SCHOLIUM.

From these propositions may be deduced an easy practical method of *Sailing by the Arc of a Great Circle*, by means of a SPECIAL CHART, on a convenient scale, of the intended tract, constructed in the following manner. (*See the* Chart.)

Having calculated by spherical trigonometry, as in the following example, the angles of position *b a* T, *a b* I, the distance *a b*, and also the latitude of the point *d* at the middle longitude; draw on a sheet of paper from a moderate scale

the line *a b* equal to the distance; and cross it at the extremities *a, b,* by the lines *a* T, *b* I; forming the angles of position at those points *b a* T, *a b* I, continued on both sides of *a b*. The latitude of the places *a* and *b*, being given, set off from the same scale of equal parts, a sufficient number of degrees of latitudes on the lines *a* T, and *b* I produced so that some common latitude shall be on both those lines, (as the latitude of 45° on the annexed *special chart)* and a perpendicular *d* T, to the middle of two points of the common latitude on the lines *a* T, *b* I will be the CENTRAL MERIDIAN, or that which passes through the middle longitude of the chart; and as the latitude of the point *d* has been found, the degrees of latitude can also be marked on the line T *d* produced on both sides of *a b*. Then the parallels of latitude will be *very nearly* represented by circles passing through the three points of each degree on the three lines *a* T, *d* T, *b* I, produced on both sides of *a b;* and dividing the extreme parallels into as many parts as there are five degrees of longitude between the two places; the several meridians can be drawn, shewing the angles of position at every five degrees of longitude, and having the same appearance as on the globe. On this SPECIAL CHART the ship's place can be marked, whenever a good observation of the latitude and longitude is taken, and a new direct course on a great circle to the intended port readily seen, with the angles of position; and the course which will meet the same great circle, after sailing 5° of longitude, can be found from the Table in page 315, by adding half the inclination of the meridians at 5 degrees difference of longitude, to the angle of position; *(the loxodromic course being always between the great circle, and the equator)* or by taking as a course, the angle of position on the special chart at 2 $\frac{1}{2}$ degrees of longitude farther onwards than the longitude of the ship; which course will serve for sailing the distance corresponding to 5° difference of longitude, (which distance can readily be found by the chart) after which the course must be altered either by adding the *whole inclination* of the meridians for 5° corresponding to the new latitude, or again taking the angle of position at 2$\frac{1}{2}$ degrees of longitude farther on, as before, which method can be continued at pleasure.

But whenever the ship is driven out of her intended course, or her latitude and longitude has been determined more correctly by astronomical observations, the *course* must be again adjusted, either by drawing a new distance-line on the Special Chart, which will shew the new angle of position to the intended port sufficiently correct, or calculated by case seven of Spherical Trigonometry, where two sides (the co-latitudes or polar distances of the two points) and the included angle (difference of longitude) are given; as in the following example, which is also necessary to shew in what manner a Special Chart is constructed, being considered as part of a great circle, containing in length the distance of the two places, with a few contiguous degrees in breadth on each side, straightened into a plane superficies or parallelogram; the meridians crossing the great circle at every five degrees of longitude, and shewing the different angles of position.

EXAMPLE.

Suppose it was intended to sail from the latitude of 40° N. and longitude 65° W. by *the arc of a great circle,* to the latitude of 49° 26′ N. and longitude of 5° W. making the nearest course to the Lizard from the above place; and to construct a *Special chart* in order to lay down the ships track. Required the angles of position, nearest distance, and the courses that will meet the great circle, at every five degrees difference of longitude?

To find the Angles of Position.

Co-lat. or Polar-dist.	50° 0′	(N. B. These two P. dists. must be from the same Pole.)		
Co-lat or Polar-dist.	40 34			
As ½ the sum of P. dists. (x)	45 17	log. sine 9.851622	log. cos. 9.847327	
: ½ the difference of ditto.	4 43	log. sine 8.915022	log. cos. 9.998527	
: : ½ the diff. of longitude	30 0	log. cot. 0.238561	log. cot. 0.238561	
: the corresponding arcs	11 20	log. tang. 9.301961	log. tan. 0.389761	
	67 49	(take the supplement of this if x exceeds 90°.)		
Sum of corresponding arcs	79 9	=Angle of Position at the greater latitude.		
Difference of ditto	56 29	=Angle of Position at the less latitude.		

To find the Distance.

As either angle of position	as	79° 9′	log. sine.	9.992166
: its opposite polar distance		50 0	log. sine.	9 884254
: : the difference of longitude		60 0	log. sine.	9.937531
: nearest distance 42° 29′=2549 miles			log. sine.	9.829619

To find the Latitude at the middle Longitude.

Latitude	40° 0′	nat. tang.	0.839100	(N. B. Should no table of nat. tang.
Latitude	49 26	nat. tang.	1.168095	be at hand, take the numbers to
				the log. tangents.)
Sum of natural tangents.			2.007195	
Half sum of ditto.			1.003597	log. 0 001558
Half the diff. of longitude		30°	subtr. log. cos.	9.937531
Gives the lat. at the middle long. or central mer.			49° 12′½ tang.	0.064027
So that the angles of position T *a b*, I *b a* are			79 9 and	56° 29′
The distance *a b*			42 29=2549 miles	
The latitude of *d*, at the middle longitude			49 12½	

From which data the *Special Chart* has been constructed according to the directions given in the Scholium, and the first course as far as 5° difference of longitude is found by adding 96′=1° 36′, (half the angle of the inclination of the meridians in the latitude 40° by the Table in page 315) to the angle of position 56° 29′; making N. 58° 5′ E. for the *course* from the longitude of 65° W. to 60° W. or if the course had been taken from the angle of position in the special chart at the longitude of $62\frac{1}{2}$° it would be the same, according to Prop. 2d. and perhaps this last is as simple a rule as can be given, for it appears to be a useless labour to calculate all the angles of position by Spherical Trigonometry, as different accidents may make the ship occasionally deviate from the intended calculated track, and a *Special Chart* will always shew the courses to sufficient exactness, if the ship were even to deviate 5 degrees of latitude on either side of the first intended track or great circle.

In the same manner a SPECIAL CHART can be constructed for any other track, by means of which it is easy to sail from any part on the globe to any other, the shortest way possible, supposing there are no intervening obstructions, or local reasons for taking a different course; the *Special Chart* being an extension of that part of the globe through which the track lies, all the bearings and distances are truly represented.

Should any intelligent navigator be inclined to try this method of sailing by the *Navigation Chart* or by the *Arc of a great Circle*, he will find these directions sufficiently correct for practice, always depending on ASTRONOMICAL OBSERVATIONS to correct his reckoning; and as the very great improvements lately made and almost universally adopted in the astronomical part of navigation, seem to require some corresponding improvements in the other parts; this attempt, the author hopes will be candidly examined.

The Loxodromic Chart from the latitude of 15° to 55° will serve where the distances greatly exceed the limits of the lesser charts, they being on a much larger scale—every 10 miles of the latter making a degree on the former, so that the same index will serve, reckoning 10 degrees for every 100 miles; and all the problems solved by Mercator's Chart, can be more readily and simply solved by the general Loxodromic Chart.

The following table, shewing the inclination of the meridians in minutes and tenths, corresponding to 5 degrees difference of longitude for every degree of latitude, is readily constructed by means of a Traverse table; using the latitude as a course, and against 300′ (the minutes in 5°) as a distance, the inclination of the meridians, as under, will be found in the column of departure.

Table of the Inclination of Meridians in Minutes and Tenths, at every Degree of Latitude for 5° Difference of Longitude.

Lat.	in. mer.	Lat.	in. mer.	Lat.	in. mer.	Lat.	in. mer.	Lat.	in. mer.	Lat.	in. mer.
°	′	°	′	°	′	°	′	°	′	°	′
1	5.2	13	67.5	25	126.8	37	180.5	49	226 4	60	259.8
2	10.5	14	72.6	26	131.5	38	184.7	50	229.8	61	262.4
3	15.7	15	77.6	27	136.2	39	188.8	51	233.1	62	264.9
4	20.9	16	82.7	28	140.8	40	192.8	52	236.4	63	267.3
5	26.1	17	87.7	29	145.4	41	196 8	53	239.6	64	269.6
6	31.4	18	92.7	30	150 0	42	200.7	54	242.7	65	271.9
7	36.6	19	97.7	31	154.5	43	204.6	55	245.7	70	281.9
8	41.8	20	102.6	32	159.0	44	208.4	56	248.7	75	289.8
9	46.9	21	107.5	33	163.4	45	212.1	57	251.6	80	295.4
10	52.1	22	112.4	34	167.8	46	215.8	58	254 4	85	298.9
11	57.2	23	117.2	35	172.1	47	219.4	59	257.2	90	300.0
12	62 4	24	122.0	36	176.3	48	222.9				

REMARK.

It appears that a general chart on this projection has all the valuable properties of Mercator's Chart; the rhumb lines and distances being right lines;

in other respects it is superior, as equal surfaces on the globe are represented by equal areas on the chart, and all distances are measured by the same scale of equal parts. It has also the advantage of shewing both the loxodromic course, and the angles of position (that is the angles made by the different meridians with the great circle passing through any two places:) the first being measured by the complement of the angle formed by the parallels of latitude and line of distance, the latter very nearly by the different meridians and line of distance; and in the great simplicity of its construction it seems also to merit the preference.

The Chart might also be enlarged so as to give any required accuracy in the solutions; but as no greater accuracy can be expected in any estimation of a ship's course and distance by the log-line and compass, (the chief dependence being on astronomical observations, for longitude as well as latitude) it would be useless.

MR. EMERSON, *in page 52 of his Geography, has also given this projection, as useful for maps; but its properties, and great use in* practical navigation, *have not, I believe, been hitherto investigated.*

TABLE I, shewing the correction in minutes and tenths, to be *added* to the Middle Latitude, in Middle Latitude Sailing.

Argument. Diff. of LAT.	ARGUMENT. LESSER LATITUDE.													
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°
°	′	′	′	′	′		′	′	′	′	′	′	′	′
1	6.7	0.8	0.1	0.1	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.2
2	9.1	1.8	0.9	0.7	0.6	0.5	0.5	0.5	0.5	0.5	0.6	0.7	0.7	0.9
3	13 9	3 5	2.1	1.6	1.3	1.2	1.1	1.1	1.1	1.2	1.3	1.5	1.7	2.0
4	18.5	5.9	3.6	2.7	2.3	2.1	2.0	2.0	2.0	2.1	2.4	2.6	3.0	3.6
5	23 3	8.5	5.4	4.1	3.5	3.2	3.1	3.1	3.2	3.4	3.7	4.1	4.8	5.7
6	28.0	11.5	7.5	5.8	5.0	4.6	4.5	4.5	4.6	4.9	5.4	6.0	7.0	8.4
7	32.9	14.8	9.9	7.8	6.8	6.3	6.1	6.1	6.3	6.7	7.4	8.3	9.7	11.8
8	37.6	18.2	12.5	10.0	8.7	8.1	8.0	8.0	8.3	8.8	9.8	11.0	12.9	15.8
9	42.3	21.9	15.4	12.4	11.0	10.2	10.0	10.1	10.5	11 3	12.5	14 2	16.6	20.5
10	47.0	25.7	18.4	15.1	13.4	12.6	12.3	12.5	13.1	14.1	15.6	17.7	20.9	26.0
11	51.8	29.8	21.7	18.0	16.1	15.2	14.9	15.2	16.0	17.2	19 1	21 8	25.9	32.4
12	56.7	34.0	25.2	21.1	19.0	18.0	17.8	18.2	19.1	20.7	23·0	26 4	31.5	39.7
13	61.6	38.3	28 9	24.4	22.1	21.1	20.9	21.4	22.6	24.5	27.4	31 5	37.8	48.1
14	66.6	42.8	32.7	27.9	25.4	24.4	24.4	25.0	26.4	28.8	32·2	37.2	45.0	57.8
15	71.6	47.3	36.7	31.6	28.9	27.9	27.9	28.8	30.6	33.4	37.5	43.6	53.0	68.8
20	97.6	72.0	59.5	53.1	50.1	49.3	50 2	52.7	56.9	63.1	72.4	86.5		
25	125.5	99.7	86.5	79.8	77.2	77.5	80.3	85.7	94.1	106.6				
30	155.9	131.2	118.1	112.1	110.8	113.4	119.6	129.9	145.5					
35	189.4	165.5	154.9	150.7	152.2	158.7	170 5	188.9						
40	226 9	206.8	197.9	197.0	202.9	215.7	236.6							
45	269.2	253.1	248.4	252.7	265.7	288.4								
50	317.7	307.0	308.4	320.6	344.5	383.7								
55	373.8	430.5	320.8	405.0	446.6	600 0								
60	439.7	446.6	470.1	513.2	586.1	720.2								

CONSTRUCTION.

Log. of diff. of lat.—log. of merid. diff. lat.=the log. cosine of the MEAN PARALLEL A.
A—middle latit=the correction of this table.

TABLE II, to calculate the exact difference between the new Loxodromic Chart and Mercator's; which in practice is nearly insensible. It also shews the error in taking the arithmetic mean of the natural cosines of two latitudes for the cosine of the mean parallel in Middle Latitude Sailing,

Argument. Diff. of LATS.	ARGUMENT. LESSER LATITUDE. 0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°
	—	—	—	—	—	—	—	—	—	—	+	+	+	+
°	′	′	′	′	′	′	′	′	′	′	′	′	′	′
1	8.1	1.0	1.1	0.6	0.3	0.2	0.1	0.1	0.1	0.0	0.1	0.1	0.2	0.3
2	22.3	6.0	3.2	2.1	1.4	1.0	0.7	0.4	0.2	0.1	0.3	0.6	0.9	1.2
3	33.0	11.2	6 6	4.4	3.1	2.2	1.5	0.9	0.6	0.2	0.7	1 3	2.0	2 9
4	44.0	17 6	10 8	7.2	5.3	3.8	2.6	1.5	0.6	0.4	1.4	2.5	3.7	5.3
5	55.0	25.0	15.8	11 0	7.9	5.6	3.8	2.2	0.7	0.7	2.3	4 0	6.0	8.6
6	66.1	32.7	21.3	15.0	10 8	7.7	5.2	2.9	0 8	1.2	3.5	6.0	8.9	12.8
7	77.2	40.9	27 2	19.3	14.0	10.0	6.6	3.6	0.8	2.0	5.1	8.4	12.6	18.1
8	88.4	49.3	33.4	24.0	17.5	12 4	8.1	4.3	0 6	3.1	7.1	11.5	16.1	24.5
9	99 6	56.9	39.8	28.9	21.1	15 0	9 7	5.0	0 4	4 3	9.4	15.2	22.4	32.2
10	109 9	66 7	46.5	34.0	24 8	18 6	11.3	5.5	0 1	5 9	12.2	19.5	28.7	41.3
15	161.6	111.2	81.7	60.7	44 4	30.4	17.8	5.6	6.7	20.0	35 1	53.5	78.1	
20	211.7	154.7	116 4	87.0	62 4	40.4	19.5	1 5	23.7	48.6	78.4	117.0		
25	259.4	195.0	148 2	109 8	75.9	44.1	12 6	20.4	56.7	99.4				
30	300.7	230.5	174 7	126.3	81.6	36.9	7.6	56.7	113.7					
35	337.4	259 6	193.7	133.9	76.1	17.2	46.2	118 7						
40	367.0	280.2	202.9	129.2	55.8	23.6	112.3							
45	387.8	290.1	199.0	108.4	13.3	92.8								
50	397.5	286.4	178.9	61 5	58.7	205.6								
55	393 2	265.0	133.9	8 9	175 8	541 0								
60	370.8	219.9	57.8	129.6	368.5	730.6								

The corrections on this side the black line must be added to the greater latitude; those on the left hand side must be subtracted.

CONSTRUCTION. { 2 Nat. cos. *mean parallel*—nat. cos. les. lat.=nat. cos. of an angle A.
A—greater latitude=the *correction* of this table.

N B. This correction × by the tangent of the greatest latitude × tangent of half the difference of longitude, is=the correction of the longitude found by the chart for any given course and distance.

USE OF THE PRECEDING TABLES.

EXAMPLE I.

Required the Course and Distance between Cape Clear in Ireland, in latitude 51° 18′ N. and Island of St. Mary's one of the Azores, in latitude 37° N. difference of latitude 14° 18′= 858 miles. and their difference of longitude being 15° 10′, or 910′. (See Robertson's Navig. prob. III. p. 156.)

Middle Latitude 44° 9′

Correction to lat. 37° and diff. of latits. 14° 18′ about 26 from Table I.

Mean Parallel 44° 35′

As difference of latitude 858′ log. 2.933487 log. A 2.933487

: Difference of longitude	910′	log.	2.959041			
: : Cos. of mean parallel 44° 35′		log. cos.	9.852620			
: Tan. of Course	37° 4′	log. tang.	9.878174	l. cos. B		9.901967
A—B=distance - -	1074,6 miles		log.			3.031520

which agrees with the solution by Meridional Parts, or logarithmic tangents; but by the middle latitude not corrected, the course would have been 37° 16′ and distance 1078 miles.

EXAMPLE II.

A ship from the latitude of 51° 18′ N. in longitude 22° 6′ W. sailing on a course between the S. and E. has made 564 miles of departure, and 786 miles difference of longitude. Required the latitude of the place arrived at? (See Robertson's Navigation, prob. X. page 170.)

					° ′
As difference of longitude	786′	log	2.895423	Mean par. × 2 =	88 18
: Departure - -	564′	log.	2.751279	Subtract given lat.	51 18
:: Radius - - -		-	10.000000		
: Cos. Mean Parallel	44° 9′ log. cos.		9.855856	Lesser lat. (nearly)	37 0
				Diff. of lat. (nearly)	14 18

As the approximate lesser latitude must be diminished by twice the correction in Table I to obtain the true lesser latitude, assume it 36° 10′, and difference of latitude 15° 8′, the correction from Table I will then be 30′;—this subtracted

from 44° 9′ the mean parallel, leaves -	43° 39′	for the Middle Latitude.
From the double of which - - -	87 18	
Subtract the greater latitude - - -	51 18	
Leaves the true lesser latitude - -	36 0	differing a whole degree from

the latitude found by the common method.

These examples sufficiently shew the great use of Table I, to correct the errors of middle latitude sailing; which by this means is made equally correct, and is more simple than Mercator's sailing by the table of meridional parts.

Table II is intended to make the Loxodromic Chart strictly accurate, although this correction in practice will be found insensible. It also shews the error of taking half the sum of the natural cosines for the cosine of the mean parallel, which has been recommended in middle latitude sailing.

This small correction of the Loxodromic Chart, by means of table II, will be easily understood from the following

EXAMPLE.

Suppose a ship to sail from the latitude of 20° N. to the latitude of 45° N. on a course between the N. and E. and make 40° difference of longitude; required the course and distance. (See the Loxodromic Chart.)

By table II, the correction for 20° lesser latitude and 25° diff. of latitudes is—75′,9, which measure off to the parallel of 45° from the meridian of 20° (the half diff. of longitude) from *a* to *b* perpendicular to the parallel, so that *a b* = 75′,9; then draw the line *c b* from the parallel of 20° latitude to the parallel of 45°, making the half difference of longitude 20° on each side the central meridian (which is essential to the principle of the chart,) and it will be the correct course and distance. The line *c d* represents the course and distance on the chart, without the tabular correction, which correction in all practical cases will be insensible.